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FLOW OF A LIQUID ABOUT A NONUNIFORMLY HEATED DROPLET WITH ARBITRARY TEMPERATURE DIFFERENCES IN ITS VICINITY

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The problem of flow of a liquid about a nonuniformly heated droplet at small Reynolds numbers is solved analytically with allowance for the effect of medium motion with arbitrary temperature differences between the particle surface and the region away from it and the temperature dependence of the viscosity, represented in the form of a series. It is shown that in the approximation considered the nonuniformly heated droplet retains a spherical shape.

1. Formulation of the Problem. Consideration is given to plane-parallel flow of a viscous liquid with velocity $\mathbf{U}_\infty$ about a sessile droplet of a liquid with viscosity μ_{liq}; the viscous liquid does not mix with the liquid of the droplet, and nonuniformly distributed sources (sinks) of heat of strength q_{p} act in it.

Unlike the flow studied earlier in [1-6], in this work consideration is given to flow about a droplet with arbitrary temperature differences between the surface of the particle and the region away from it with allowance for internal heat sources nonuniformly distributed throughout the volume of the particle with density q_{p}. The temperature dependence of the viscosity, which is represented in the form of a series, is taken into account in the hydrodynamic equations, while the convective terms are allowed for in the heat-conduction equation.

Oseen [7] and Praudman and Pearson [8] have shown for the hydrodynamic problem, and Acrivos and Taylor [9] for the heat problem, that far from a sphere the inertial and convective terms become of the same order as the terms of molecular transfer, and therefore the conventional method of expansion in a small parameter yields a known error, since even in a second approximation it is impossible to rigorously satisfy the boundary conditions at infinity or to obtain a single exact solution that holds uniformly for the entire region of the flow.

The present work seeks to determine, for this case, the velocity field and an expression for the resistance force that acts on a nonuniformly heated droplet with arbitrary temperature differences in its vicinity.

It is assumed that the densities, thermal conductivities, and heat capacities of the liquids are constant outside and inside the droplet, the coefficient of surface tension is an arbitrary function of the temperature ($\sigma = \sigma(T)$), and the coefficient of thermal conductivity of the droplet is much larger in magnitude than the coefficient of thermal conductivity of the surrounding liquid. Flow about the droplet is rather slow (a small Reynolds number), and it retains a spherical shape (distortion of the shape will be considered below).

The presence of heat sources (sinks) inside the droplet leads to the fact that the average temperature of its surface can differ significantly from the temperature of the surrounding liquid away from it. The heating of the droplet surface has an effect on the thermophysical characteristics of the surrounding liquid and ultimately on the distribution of the velocity and pressure fields in its vicinity. Of all the parameters of liquid transfer, only the coefficient of viscosity depends strongly on the temperature [10]. Therefore we use a formula that enables us to describe the viscosity variation in a wide temperature interval with any required degree of accuracy (when $F_n = 0$ this formula can be reduced to the well-known Reynolds relation [10]):

$$\mu_{\text{liq}} = \mu_\infty \left[1 + \sum_{n=1}^{\infty} F_n \left(\frac{T_{\text{liq}}}{T_\infty} - 1 \right)^n \right] \exp\left\{ -A \left(\frac{T_{\text{liq}}}{T_\infty} - 1 \right) \right\}. \tag{1.1}$$

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The liquid viscosity is known to decrease by an exponential law as the temperature increases [10]. An analysis of the available semiempirical formulas has shown that expression (1.1) makes it possible to describe in the best manner the viscosity variation in a wide temperature interval with any required degree of accuracy.

The origin of a spherical coordinate system r, θ, φ is selected at the center of the droplet. Within the framework of the assumptions formulated the equations and boundary conditions for the velocity and temperature will be written in the spherical coordinate system in the form [11, 12]

$$\frac{\partial P_{\text{liq}}}{\partial x_k}=\frac{\partial}{\partial x_j}\left\{\mu_{\text{liq}}\left(\frac{\partial U_k^{\text{liq}}}{\partial x_j}+\frac{\partial U_j^{\text{liq}}}{\partial x_k}\right)\right\},\quad \operatorname{div}\mathbf{U}_{\text{liq}}=0\,; \tag{1.2}$$

$$\mu_{\text{p}}\Delta\mathbf{U}_{\text{p}}=\nabla P_{\text{p}},\quad \operatorname{div}\mathbf{U}_{\text{p}}=0\,; \tag{1.3}$$

$$\rho_{\text{liq}}c_p\,(\mathbf{U}_{\text{liq}}\cdot\nabla)\,T_{\text{liq}}=\lambda_{\text{liq}}\Delta T_{\text{liq}},\quad \Delta T_{\text{p}}=-\,q_{\text{p}}/\lambda_{\text{p}}\,; \tag{1.4}$$

$$r=R,\quad T_{\text{liq}}=T_{\text{p}},\quad \lambda_{\text{liq}}\frac{\partial T_{\text{liq}}}{\partial r}=\lambda_{\text{p}}\frac{\partial T_{\text{p}}}{\partial r},\quad U_r^{\text{liq}}=U_r^{\text{p}}=0,\quad U_\theta^{\text{liq}}=U_\theta^{\text{p}}\,; \tag{1.5}$$

$$\mu_{\text{liq}}\left[r\frac{\partial}{\partial r}\left(\frac{U_\theta^{\text{liq}}}{r}\right)+\frac{1}{r}\frac{\partial U_r^{\text{liq}}}{\partial\theta}\right]+\frac{1}{r}\frac{\partial\sigma}{\partial T_{\text{p}}}\frac{\partial T_{\text{p}}}{\partial\theta}=\mu_{\text{p}}\left[r\frac{\partial}{\partial r}\left(\frac{U_\theta^{\text{p}}}{r}\right)+\frac{1}{r}\frac{\partial U_r^{\text{p}}}{\partial\theta}\right];$$

$$r\to\infty,\quad \mathbf{U}_{\text{liq}}\to U_\infty\cos\theta\,\mathbf{e}_r-U_\infty\sin\theta\,\mathbf{e}_\theta,\quad P_{\text{liq}}\to P_\infty,\quad T_{\text{liq}}\to T_\infty\,; \tag{1.6}$$

$$r\to 0,\quad |\mathbf{U}_{\text{p}}|\neq\infty,\quad P_{\text{p}}\neq\infty,\quad T_{\text{p}}\neq\infty\,. \tag{1.7}$$

When $\lambda_{\text{p}}>>\lambda_{\text{liq}}$ the flow about the droplet occurs with small temperature differences in its volume. In this connection, the coefficient of viscosity of the droplet will be considered to be a constant (which is allowed for in Eq. (1.3)).

In the boundary conditions (1.5) on the surface of the droplet ($r = R$) we allowed for the impermeability condition for normal velocity components, the equality of temperatures, the continuity of heat fluxes, the equality of tangential velocities for the internal and external media, and the continuity of tangential components of the stress tensor.

The boundary conditions (1.6) hold at a large distance from the droplet ($r\to\infty$), and the finiteness of the physical quantities that characterize the particle when $r\to 0$ is allowed for in (1.7).

We make Eqs. (1.2)-(1.4) and the boundary conditions (1.5)-(1.7) dimensionless by introducing the dimensionless coordinate, velocity, and temperature in the following manner: $y_k = x_k/R$, $t = T/T_\infty$, and $\mathbf{V} = \mathbf{U}/U_\infty$.

At $\text{Re}_\infty = (\rho_{\text{liq}}U_\infty R)/\mu_\infty << 1$ the incoming flow has only a perturbing effect, and therefore the solution of the hydrodynamic equations should be represented in the form

$$\mathbf{V}=\mathbf{V}^{(0)}+\varepsilon\,\mathbf{V}^{(1)}+\ldots\quad(\varepsilon=\text{Re}_\infty)\,. \tag{1.8}$$

The solution of the equation that describes the temperature distribution outside the droplet will be sought by the method of joined asymptotic expansions [13]. The internal and external asymptotic expansions of the dimensionless temperature are represented as

$$t_{\text{liq}}(y,\theta)=\sum_{n=0}^{\infty}f_n(\varepsilon)\,t_{en}(y,\theta)\,,\quad f_0(\varepsilon)=1\,, \tag{1.9}$$

$$\overset{*}{t}_{\text{liq}}(\xi,\theta)=\sum_{n=0}^{\infty} f_n^*(\varepsilon)\,\overset{*}{t}_{en}(\xi,\theta)\,, \tag{1.10}$$

where $\xi = \varepsilon y$ is the "contracted" radial coordinate [13]. It is required that

$$\frac{f_{n+1}}{f_n}\to 0\,,\quad \frac{f^*_{n+1}}{f^*_n}\to 0 \quad \text{when} \quad \varepsilon\to 0\,. \tag{1.11}$$

The boundary conditions lacking for the internal and external expansions follow from the condition of identity of their asymptotic extensions into a certain intermediate region

$$t_{\text{liq}}(y\to\infty,\theta)=\overset{*}{t}_{\text{liq}}(\xi\to 0,\theta)\,. \tag{1.12}$$

The asymptotic expansion of the solution inside the particle, as the boundary conditions on the droplet surface (1.5) show, should be sought in a form that is similar to (1.9):

$$t_{\text{p}}(y,\theta)=\sum_{n=0}^{\infty} f_n(\varepsilon)\,t_{in}(y,\theta)\,. \tag{1.13}$$

For the functions $f_n(\varepsilon)$ and $f_n^*(\varepsilon)$, it is assumed only that the order of their smallness with respect to ε increases with n.

With allowance for the contracted radial coordinate we have the following equation for the temperature $\overset{*}{t}_{\text{liq}}$:

$$\text{Pr}\left(V_r^*\frac{\partial \overset{*}{t}_{\text{liq}}}{\partial\xi}+\frac{V_\theta^*}{\xi}\frac{\partial \overset{*}{t}_{\text{liq}}}{\partial\theta}\right)=\Delta^*\overset{*}{t}_{\text{liq}}\,,\quad \overset{*}{t}_{\text{liq}}\to 1 \quad \text{when} \quad \xi\to\infty\,, \tag{1.14}$$

and

$$\mathbf{V}^*_{\text{liq}}(\xi,\theta)=\mathbf{e}_z+\varepsilon\,\mathbf{V}^{(1)*}_{\text{liq}}(\xi,\theta)+\dots\,. \tag{1.15}$$

Here Δ^* is the axisymmetric Laplace operator obtained from Δ by substituting ξ for y, $V_r^* = V_r^*(\xi, \theta)$, $V_\theta^* = V_\theta^*(\xi, \theta)$, $\overset{*}{t}_{\text{liq}} = \overset{*}{t}_{\text{liq}}(\xi, \theta)$, $\text{Pr} = \mu_\infty c_p/\lambda_{\text{liq}}$ is the Prandtl number, and $\mathbf{e}_z$ is the unit vector in the direction of the z axis.

The form of the boundary conditions (1.5)-(1.7) indicates that the expressions for the velocity components V_r and V_θ are sought in the form of expansions in Legendre and Gegenbauer polynomials [11, 12]. As is shown in [12], the force that acts on the particle is determined by the first terms of these expansions; therefore we can write

$$V_r=G(y)\cos\theta\,,\quad V_\theta=-g(y)\sin\theta\,, \tag{1.16}$$

where $G(y)$ and $g(y)$ are arbitrary functions that depend on the radial coordinate y.

In investigating flow about nonuniformly heated droplets in a viscous medium we must take into account the relation of the coefficient of surface tension to the temperature in addition to the dependence of the coefficient of dynamic viscosity on the temperature. This is caused by the nonuniformity of the distribution of the density of the heat sources q_{p} in the droplet volume. In the work, an arbitrary form of the dependence of the coefficient of surface tension on the temperature was used. Furthermore, for the first time an attempt was made to allow for the effect of the medium motion on the resistance force of a heated droplet in a viscous liquid. Therefore the formulas ultimately obtained are most general in character and hold for any temperature differences between the particle surface and the region away from it.

2. Temperature Fields outside and inside the Heated Droplet. In finding the force that acts on the nonuniformly heated droplet and the velocity of its motion, we restrict ourselves to corrections of first order of smallness. To find them, we must know the temperature fields outside and inside the droplet. For this purpose, we must solve Eqs. (1.4) with the corresponding boundary conditions.

The construction of the solution begins with determination of the zero term of the external expansion (1.10). In this case, the problem is obviously satisfied by the solution

$$\overset{*}{t}_{e0} = 1\,. \tag{2.1}$$

We find the zero term of the internal expansion (1.9). When $\varepsilon = 0$ we have

$$\Delta t_{e0} = 0 \tag{2.2}$$

with the boundary conditions

$$t_{e0} = t_{i0}\,,\quad \lambda_{\text{liq}} \frac{\partial t_{e0}}{\partial y} = \lambda_{\text{p}} \frac{\partial t_{i0}}{\partial y} \quad \text{when} \quad y = 1\,. \tag{2.3}$$

The general solution of Eq. (2.2) that satisfies the boundary condition (2.3) has the form

$$t_{e0} = \Gamma_0 + \frac{\gamma}{y} + \sum_{n=1}^{\infty} \left(\frac{\Gamma_n}{y^{n+1}} + \gamma_n y^n \right) P_n (\cos\theta)\,. \tag{2.4}$$

Here γ, γ_n, and Γ_n are integration constants; $P_n(\cos\theta)$ are Legendre polynomials [14].

We determine the integration constants γ, γ_n, and Γ_n from the condition of joining, for which the external solution must be expanded into a series in ξ. Then the values of the constants are established from the requirement of correspondence of the behavior of the terms of the obtained series for $\xi \to 0$ to the behavior of the terms of expansion (24) for $y \to \infty$. For the zero approximations, the joining is trivial; we obtain $\Gamma_0 = 1$ and $\Gamma_n = \gamma_n = 0$ ($n = 1, 2, \ldots$). Consequently

$$t_{e0}(y) = 1 + \frac{\gamma}{y}\,. \tag{2.5}$$

In the subsequent solution of the problem, we must know the temperature field inside the particle. Substituting (1.13) into the second equation of (1.4), we obtain the following general solution for $t_{\text{p}}(y, \theta)$ that satisfies the finiteness of the solution when $y \to 0$:

$$t_{\text{p}}(y, \theta) = \sum_{n=0}^{\infty} \varepsilon^n t_{in}(y)\, P_n(\cos\theta)\,, \tag{2.6}$$

where $t_{in}(y)$ is a function that depends on the radial coordinate and has the form

$$t_{in}(y) = B_n y^n + \frac{1}{(2n+1)\, y^{n+1}} \int_0^1 \psi_n(y)\, y^n dy + \\ + \frac{1}{2n+1} \left[y^n \int_1^y \frac{\psi_n(y)}{y^{n+1}} dy - \frac{1}{y^{n+1}} \int_1^y \psi_n(y)\, y^n dy \right]. \tag{2.7}$$

Here

$$\psi_n(y) = -\frac{R^2}{\lambda_p T_\infty} y^2 \frac{2n+1}{2} \int\limits_{-1}^{+1} q_p P_n(\cos\theta)\, d(\cos\theta)\,.$$

In what follows we will need the expressions for the functions $t_{i0}(y)$ and $t_{i1}(y)$:

$$t_{i0}(y) = B_0 + \frac{1}{4\pi R T_\infty \lambda_p y} \int\limits_V q_p dV + \int\limits_1^y \frac{\psi_0}{y} dy - \frac{1}{y} \int\limits_1^y \psi_0 dy\,, \tag{2.8}$$

$$t_{i1}(y) = B_1 y + \frac{1}{4\pi R^2 T_\infty \lambda_p y^2} \int\limits_V q_p z dV + \frac{1}{3}\left[y \int\limits_1^y \frac{\psi_1}{y^2} dy - \frac{1}{y^2} \int\limits_1^y \psi_1 y dy \right]. \tag{2.9}$$

In (2.8) and (2.9) the integration is carried out over the entire volume of the heated droplet, and $z = r\cos\theta$.

Since the temperature field inside the nonuniformly heated droplet is determined, we can find the integration constants γ and B_0. We obtain the constants γ and B_0 that enter in (2.5) and (2.8) from the boundary conditions on the droplet surface (1.5). In our case, they will be written in the form

$$y = 1\,, \quad t_{e0} = t_{i0}\,, \quad \lambda_{liq} \frac{\partial t_{e0}}{\partial y} = \lambda_p \frac{\partial t_{i0}}{\partial y}\,. \tag{2.10}$$

Having substituted expressions (2.5) and (2.8) into (2.10), we have

$$\gamma = t_s - 1\,, \quad B_0 = \left(1 - \frac{\lambda_{liq}}{\lambda_p}\right)(t_s - 1) + 1\,.$$

Here $t_s = T_s/T_\infty$, T_s is the average temperature of the surface of the heated droplet, determined by the formula

$$\frac{T_s}{T_\infty} = 1 + \frac{1}{4\pi R \lambda_{liq} T_\infty} \int\limits_V q_p dV\,. \tag{2.11}$$

In (2.11), the integration is carried out over the entire volume of the heated droplet.

When $\lambda_{liq} << \lambda_p$, in the coefficient of dynamic viscosity we can disregard the dependence on the angle θ in the system of droplet–liquid medium and consider that the viscosity is related only to the temperature $t_{e0}(y)$, i.e., $\mu_{liq}(t_{liq}) = \mu_{liq}(t_{e0})$. With allowance for this, expression (1.1) acquires the form

$$\mu_{liq} = \mu_\infty \exp\left\{-A \frac{\gamma}{y}\right\}\left[1 + \sum_{n=1}^{\infty} F_n \frac{\gamma^n}{y^n}\right]. \tag{2.12}$$

In what follows, formula (2.12) is employed to find the velocity and pressure fields in the vicinity of the heated droplet.

Up to terms of the first approximation of the external expansion, from (1.10) we have

$$t^*_{liq}(\xi, \theta) = 1 + f^*_1(\varepsilon)\, t^*_{e1}(\xi, \theta)\,.$$

It is seen that, to find the first approximation of the external expansion, we must first determine the explicit form of the coefficient $f^*_1(\varepsilon)$. For this purpose, in solution (2.5) we pass to the external variable ξ. Then it follows from (2.5) that, for $f^*_1(\varepsilon) = \varepsilon$,

$$t^*_{\text{liq}}(\xi,\theta) = 1 + \varepsilon\, t^*_{\text{e}1}(\xi,\theta)\,. \tag{2.13}$$

Substituting (2.13) into (1.14), allowing for (1.15), and retaining terms of the order of ε, we obtain

$$\Lambda t^*_{\text{e}1} = 0\,,\quad \Lambda = \Delta^* - \Pr\left(x\frac{\partial}{\partial\xi} + \frac{1-x^2}{\xi}\frac{\partial}{\partial x}\right),\quad x = \cos\theta\,,$$
$$\xi \to \infty\,,\quad t^*_{\text{e}1} \to 0\,. \tag{2.14}$$

The general solution of Eq. (2.14) has the form [15]

$$t^*_{\text{e}1} = \exp\left\{\frac{\Pr}{2}\xi x\right\}\left(\frac{\pi}{\Pr\xi}\right)^{1/2}\sum_{n=0}^{\infty} L_n K_{n+1/2}\left(\frac{\Pr\xi}{2}\right)P_n(x)\,, \tag{2.15}$$

$$K_{n+1/2}\left(\frac{\Pr\xi}{2}\right) = \left(\frac{\pi}{\Pr\xi}\right)^{1+1/2}\exp\left\{-\frac{\Pr\xi}{2}\right\}\sum_{m=0}^{n}\frac{(n+m)\,!}{(n-m)\,!\,m\,!\,(\Pr\xi)^m}\,.$$

Here $K_{n+1/2}(\Pr\xi/2)$ is a modified Bessel function [15]. The arbitrary integration constants L_n must be determined as a result of joining, which, in this case, consists in comparing the behavior of the function (2.13) for $\xi \to 0$ and the behavior of the function (2.5) for $y \to \infty$. It is easy to establish that $L_0 = \gamma\Pr/\pi$, $L_n = 0$ for $n = 1, 2, \ldots$. Consequently

$$t^*_{\text{e}1}(\xi,\theta) = \frac{\gamma}{\xi}\exp\left\{\frac{1}{2}\Pr\xi\,(x-1)\right\}. \tag{2.16}$$

We find the first approximation for the internal expansion. From (2.16) it is seen that $f_1(\varepsilon) = \varepsilon$. Thus, we have the two-term internal expansion

$$t_{\text{liq}}(y,\theta) = t_{\text{e}0}(y) + \varepsilon\, t_{\text{e}1}(y,\theta)\,. \tag{2.17}$$

For $t_{\text{e}1}$ and $t_{\text{i}1}$ in the two-term internal expansion we obtain from (1.4)-(1.7) the following problem:

$$\Pr V_r^{\text{liq}}\frac{\partial t_{\text{e}0}}{\partial y} = \Delta t_{\text{e}1}\,,$$

$$t_{\text{i}1}(y) = B_1 y + \frac{1}{4\pi R^2 T_\infty \lambda_{\text{p}} y^2}\int_V q_{\text{p}} z\, dV + \frac{1}{3}\left[y\int_1^y \frac{\psi_1}{y^2}\,dy - \frac{1}{y^2}\int_1^y \psi_1 y\, dy\right], \tag{2.18}$$

$$y = 1\,,\quad t_{\text{e}1} = t_{\text{i}1}\,,\quad \lambda_{\text{liq}}\frac{\partial t_{\text{e}1}}{\partial y} = \lambda_{\text{p}}\frac{\partial t_{\text{i}1}}{\partial y}\,. \tag{2.19}$$

To determine the behavior of $t_{\text{e}1}(\infty,\theta)$, we join the two-term internal and external expansions

$$t_{\text{liq}}(y,\theta) = t_{\text{e}0}(y) + \varepsilon\, t_{\text{e}1}(y,\theta)\,,\quad t^*_{\text{liq}}(\xi,\theta) = 1 + \varepsilon\frac{\gamma}{\xi}\exp\left\{\frac{1}{2}\Pr\xi\,(x-1)\right\},$$

and as a result we have

$$t_{e1}(\infty, \theta) = \frac{\omega}{2}(\cos\theta - 1), \quad \omega = \Pr\gamma. \tag{2.20}$$

From (2.18) we see that, to find t_{e1}, we must first determine the velocity field, i.e., solve the hydrodynamic problem.

3. Determination of the Resistance Force. Substituting (2.12) into the hydrodynamic equation, allowing for (1.16), and separating variables, we obtain an equation that is similar to [16]. Ultimately, we have the following expressions for the components of the mass velocity and the pressure:

$$V_r^{\text{liq}}(y, \theta) = \cos\theta\,(1 + A_1G_1 + A_1G_2), \quad V_\theta^{\text{liq}}(y, \theta) = -\sin\theta\,(1 + A_1G_3 + A_2G_4),$$

$$p_{\text{liq}}(y, \theta) = 1 + \eta_{\text{liq}}\cos\theta\,(A_1G_5 + A_2G_6), \tag{3.1}$$

$$V_r^{\text{p}}(y, \theta) = \cos\theta\,(A_3 + A_4y^2), \quad V_\theta^{\text{p}}(y, \theta) = -\sin\theta(A_3 + 2A_4y^2),$$

$$p_{\text{p}}(y, \theta) = p_0 + 10\eta_{\text{p}}\cos\theta\, y^2A_4, \quad \eta = \mu/\mu_\infty, \tag{3.2}$$

where

$$G_1 = -\frac{1}{y^3}\sum_{n=0}^{\infty}\frac{\Delta_n^{(1)}}{(n+3)\,y^n}; \quad G_3 = G_1 + \frac{y}{2}G_1^{\text{I}}; \quad G_4 = G_2 + \frac{y}{2}G_2^{\text{I}};$$

$$G_2 = -\frac{1}{y}\sum_{n=0}^{\infty}\frac{\Delta_n^{(2)}}{(n+1)\,y^n} - \frac{\alpha}{y^3}\sum_{n=0}^{\infty}\left[(n+3)\ln\frac{1}{y} - 1\right]\frac{\Delta_n^{(1)}}{(n+3)^2\,y^n};$$

$$G_5 = \frac{y^2}{2}G_1^{\text{III}} + y\left(3 + \frac{1}{2}\sum_{n=0}^{\infty}s_n\frac{\gamma^n}{y^n}\right)G_1^{\text{II}} + \left(2 + \sum_{n=0}^{\infty}s_n\frac{\gamma^n}{y^n}\right)G_1^{\text{I}};$$

$$G_6 = \frac{y^2}{2}G_2^{\text{III}} + y\left(3 + \frac{1}{2}\sum_{n=0}^{\infty}s_n\frac{\gamma^n}{y^n}\right)G_2^{\text{II}} + \left(2 + \sum_{n=0}^{\infty}s_n\frac{\gamma^n}{y^n}\right)G_2^{\text{I}}, \tag{3.3}$$

$$s_n = AF_{n-1} - nF_n - \sum_{k=1}^{n}s_{n-k}F_k,$$

$F_0 = 1$, F_n is equal to zero for $n < 0$.

In (3.3), G_k^{I}, G_k^{II}, and G_k^{III} are the first, second, and third derivatives of the corresponding functions with respect to y ($k = 1, 2$). The values of the coefficients $\Delta_n^{(1)}$ and $\Delta_n^{(2)}$ are found using the following recurrence relations:

$$\Delta_n^{(1)} = -\frac{1}{n(n+5)}\sum_{k=1}^{n}\left[(n+4-k)\left\{\alpha_k^{(1)}(n+5-k) - \alpha_k^{(2)}\right\} + \alpha_k^{(3)}\right]\gamma^k\Delta_{n-k}^{(1)},$$

$$n \geq 1; \tag{3.4}$$

$$\Delta_n^{(2)} = -\frac{1}{(n+3)(n-2)} \Bigg[6\alpha_n^{(4)} \gamma^n + \sum_{k=1}^{n} \Big\{ (n+2-k) \left[(n+3-k)\, \alpha_k^{(1)} - \alpha_k^{(2)} \right] +$$

$$+ \alpha_k^{(3)} \Big\} \gamma^k \Delta_{n-k}^{(2)} + \alpha \sum_{k=0}^{n} \left\{ (2n+5-2k)\, \alpha_k^{(1)} - \alpha_k^2 \right\} \gamma^k \Delta_{n-k-2}^{(1)} \Bigg], \quad n \geq 3. \tag{3.5}$$

In calculating the coefficients $\Delta_n^{(1)}$ and $\Delta_n^{(2)}$ by formulas (3.4) and (3.5) it must be taken into account that $\Delta_0^{(1)} = \Delta_0^{(2)} = 1$, $\Delta_2^{(2)} = 1$, $\alpha_0^{(2)} = 4$, $\alpha_0^{(1)} = \alpha_0^{(4)} = 1$, $\alpha_0^{(3)} = -4$, $\alpha_n^{(1)} = F_n$, $\alpha_n^{(2)} = (4-n)F_n + AF_{n-1}$, $\alpha_n^{(4)} = A^n/n!$, $\alpha = -6\gamma^2 \alpha_2^{(4)}/5$, $\alpha_n^{(3)} = 2AF_{n-1} - 2(2+n)F_n$, $\Delta_1^2 = \gamma[6\alpha_1^{(4)} + 2(3\alpha_1^{(4)} + 2(3\alpha_1^{(1)} - \alpha_1^{(2)}) + \alpha_1^{(3)}]/4$.

Substituting (3.1) into (2.18), we obtain the following equation for the function t_{e1}:

$$\Delta t_{e1} = -\frac{\omega}{y^2} G(y) \cos\theta. \tag{3.6}$$

Here $G(y) = 1 + A_1 G_1 + A_2 G_2$ and $\omega = \Pr\gamma$.

The solution for t_{e1} is sought in the form

$$t_{e1} = \zeta(y) + \tau_e(y) \cos\theta, \tag{3.7}$$

with the boundary conditions

$$\zeta(y) \to -\frac{\omega}{2}, \quad \tau_e(y) \to \frac{\omega}{2} \quad \text{when} \quad y \to \infty, \tag{3.8}$$

$$\zeta = 0, \quad \tau_e = \text{const} \quad \text{when} \quad y = 1.$$

Substituting (3.7) into (3.6), we assure ourselves that the variables are separated and

$$\zeta(y) = \frac{\omega}{2y}(1-y),$$

while τ_e satisfies the equation

$$\frac{d^2\tau_e}{dy^2} + \frac{2}{y}\frac{d\tau_e}{dy} - \frac{2}{y^2}\tau_e = -\frac{\omega}{y^2} G. \tag{3.9}$$

The general solution of Eq. (3.9) that satisfies the boundary conditions (3.8) has the form

$$t_{e1}(y, \theta) = \frac{\omega}{2y}(1-y) + \left\{ \frac{\Gamma}{y^2} + \omega \sum_{k=1}^{3} A_k \tau_k \right\} \cos\theta, \tag{3.10}$$

where

$$\tau_1(y) = \frac{1}{y^3} \sum_{n=0}^{\infty} \frac{\Delta_n^{(1)}}{(n+1)(n+3)(n+4)\, y^n};$$

$$\tau_2(y) = -\frac{1}{y} \left\{ \frac{1}{2} + \frac{\Delta_1^{(2)}}{6y} \ln y - \sum_{n=2}^{\infty} \frac{\Delta_n^{(2)}}{(n^2-1)(n+2)\, y^n} - \frac{\alpha}{y^2} \times \right.$$

$$\times \sum_{n=0}^{\infty} [(n+1)(n+3)(n+4)\ln\frac{1}{y} - 3n^2 - 16n - 19] \frac{\Delta_n^{(1)}}{(n+1)^2 (n+3)^2 (n+4)^2 y^n}\Bigg\} ,$$

$$\tau_3 = \frac{1}{2}, \quad A_3 = 1 .$$

The integration constant Γ is determined from the boundary conditions on the droplet surface (2.19) and is equal to

$$\Gamma = \frac{1}{\delta}\left\{\frac{3}{4\pi R^2 \lambda_p T_\infty}\int_V q_p z dV - \omega \sum_{k=1}^{3} A_k \varphi_k\right\} .$$

Here $\delta = 1 + 2\frac{\lambda_{liq}}{\lambda_p}$ and $\varphi = \tau_k - \frac{\lambda_{liq}}{\lambda_p}\tau_k^{I}$; the superscript I denotes the first derivative with respect to y.

Thus, in a first approximation with respect to ε, we determined the temperature fields outside and inside the nonuniformly heated droplet. Consequently, by employing the boundary conditions on the particle surface for the velocity components we can find the integration constants A_1, A_2, A_3, and A_4 that enter in expressions (3.1) and (3.2). After calculating them the force that acts on the nonuniformly heated droplet is determined by integration of the stress tensor over the particle surface [12], and it will be comprised additively of the force of viscous resistance of the medium $\mathbf{F}_\mu$, a force $\mathbf{F}_q$ that is proportional to the dipole moment of the heat-source density, and the force $\mathbf{F}_d$ that is caused by the motion of the medium:

$$\mathbf{F} = \mathbf{F}_\mu + \varepsilon \mathbf{F}^{(1)}, \quad \mathbf{F}^{(1)} = \mathbf{F}_q + \mathbf{F}_d , \tag{3.11}$$

where $\mathbf{F}_\mu = -6\pi R \mu_\infty U_\infty f_\mu \mathbf{e}_z$; $\mathbf{F}_q = 6\pi R \mu_\infty f_q J \mathbf{e}_z$; $\mathbf{F}_d = 6\pi R \mu_\infty f_d \mathbf{e}_z$.

The coefficients f_μ, f_q, and f_d can be evaluated from the following expressions:

$$f_\mu = \frac{2}{3\Delta}\exp(-A\gamma)\left(N_3 + \frac{\mu_{liq}^s}{3\mu_p^s} N_4\right), \quad \delta = 1 + 2\frac{\lambda_{liq}^s}{\lambda_p^s}, \quad V = \frac{4}{3}\pi R^3 ,$$

$$\Delta = N_1 + \frac{\mu_{liq}^s}{3\mu_p^s} N_2 - \frac{2\rho_{liq} R}{3\mu_p^s}\frac{\omega}{\delta\mu_\infty}\frac{\lambda_{liq}^s}{\lambda_p^s}(G_1\Phi_2 - G_2\Phi_1)\frac{\partial\sigma}{\partial t_p} ,$$

$$f_q = \frac{4R}{9\mu_p^s \Delta}\exp\{-A\gamma\}\frac{G_1}{\lambda_p^s T_\infty \delta}\frac{\partial\sigma}{\partial t_p}, \quad z = r\cos\theta , \tag{3.12}$$

$$f_d = \frac{4}{9\mu_p^s \Delta} G_1 \exp\{-A\gamma\}\frac{\lambda_{liq}^s}{\delta\lambda_p^s}\omega\left(1 - \frac{\Phi_1}{G_1}\right)\frac{\partial\sigma}{\partial t_i}, \quad \Phi_k = 2\tau_k + \tau_k^{I}, \quad k = 1, 2 ,$$

$\mathbf{e}_z$ is the unit vector in the direction of the z axis, and $J = \frac{1}{V}\int_V q_p z dV$ is the dipole moment of the heat-source density.

In evaluating the coefficients f_μ, f_q, and f_d, it must be taken into account that the superscript s denotes physical quantities taken at the average temperature of the droplet surface T_s, which is determined by formula (2.15); the functions Φ_1, Φ_2, G_1, G_2, N_1, N_2, N_3, and N_4 are taken at $y = 1$ ($N_4 = 2G_1^{I} + G_1^{II}$, $N_1 = G_1 G_2^{I} - G_2 G_1^{I}$, $N_3 = -G_1^{I}$, $N_2 = G_2(2G_1^{I} + G_1^{II}) - G_1(2G_2^{I} + G_2^{II})$).

In the case where the heating of the droplet surface is rather small, i.e., the average temperature of the droplet surface differs slightly from the temperature of the surrounding medium at infinity ($\gamma \to 0$), the dependence of the coefficient of viscosity on the temperature can be disregarded, and then $G_1 = -1/3$, $G_1^{\mathrm{I}} = 1$, $G_1^{\mathrm{II}} = -4$, $G_2 = -1$, $G_2^{\mathrm{I}} = -1$, $G_2^{\mathrm{II}} = -2$, $N_1 = 2/3$, $N_2 = 2$, $N_3 = -1$, and $N_4 = -2$.

The first formula of (3.11) enables us, with the known volume distribution of the heat sources, to take into account the effect of the medium motion on the resistance force that acts on the nonuniformly heated droplet with arbitrary temperature differences between the particle surface and the region away from it with allowance for the exponential form of the temperature dependence of the viscosity. The above formulas (3.11) are the most general in character.

It is also seen from (3.11) that the magnitude and direction of the total resistance force $\mathbf{F}^{(1)}$ will be affected by the magnitude and direction of the dipole moment of the heat-source density.

If, for example, the droplet surface is heated due to absorption of electromagnetic radiation, the dipole moment can be both negative (most of the heat energy is released in the portion of the particle that faces the radiation flux) and positive (most of the heat energy is released in the shadow portion of the particle). This depends on the optical properties of the droplet. Since the surface tension for the majority of liquids decreases with the temperature, i.e., $\partial\sigma/\partial t_p < 0$, and the value of J can be both positive and negative, the magnitude of the total resistance force $\mathbf{F}^{(1)}$ will also change.

Furthermore, from the formulas obtained it is seen that this force depends substantially on the thermal conductivity of the droplet. When λ_p tends to infinity the resistance force tends to zero for a fixed magnitude of the dipole moment of the heat-source density.

4. Distortion of the Shape of the Surface. The shape of the droplet surface is not known in advance and must be determined from the solution, and therefore the boundary conditions (1.5)-(1.7) for the problem in question are set on an unknown boundary. Since we restrict ourselves to corrections of first order of smallness in ε, we can write

$$\sigma = \sigma_0 + \varepsilon\sigma^{(1)}, \tag{4.1}$$

where σ_0 is the zero term in the expansion of the function $\sigma(x)$ in Legendre polynomials $P_n(x)$, $x = \cos\theta$.

The shape of the droplet surface is sought in the form [11]

$$r = R\,(1 + \varepsilon\xi)\,. \tag{4.2}$$

We expand the sought quantities $\sigma^{(1)}(x)$ and $\xi(x)$ in series in Legendre polynomials:

$$\sigma = \sum_{n=0}^{\infty} \sigma_n P_n(\cos\theta)\,, \quad \xi = \sum_{n=0}^{\infty} \xi_n P_n(\cos\theta)\,. \tag{4.3}$$

From the condition of constancy of the droplet volume it follows that $\xi_0 = 0$. Since the origin of the coordinate system is located at the center of mass of the heated particle, then

$$\int_0^{\pi} \xi \sin^2\theta d\theta = 0 \tag{4.4}$$

and, consequently,

$$\xi_1 \equiv 0\,. \tag{4.5}$$

In solving the problem, we did not allow for the boundary condition for the normal components of the stress tensor. Accurate to terms that are proportional to ε, the boundary condition for the normal stresses on the droplet surface can be written in the form [12]

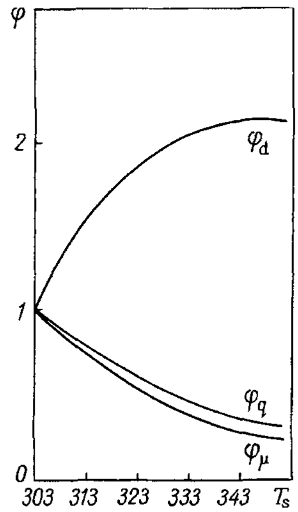

Fig. 1. Coefficients $\varphi_\mu = f_\mu/f_\mu^*$, $\varphi_q = f_q/f_q^*$, and $\varphi_d = f_d/f_d^*$ vs. average temperature of the particle surface T_s.

$$\sigma_n^{\mathrm{liq}(1)} - \sigma_n^{\mathrm{p}(1)} = \sigma_0 H^{(1)} + 2\frac{\sigma^{(1)}}{R}. \tag{4.6}$$

Here $2H = \frac{1}{R_1} + \frac{1}{R_2} = \frac{2}{R} + \varepsilon H^{(1)}$, R_1 and R_2 are the principal radii of curvature of the droplet surface; H is the average curvature of the surface, which, in the axisymmetric case, is equal to [12]

$$H^{(1)} = -\frac{2}{R}\xi - \frac{1}{R\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\xi}{\partial\theta}\right). \tag{4.7}$$

Using (4.3) and (4.5), we represent expression (4.7) as

$$H^{(1)} = \sum_{n=2}^{\infty} \frac{(n+2)(n-1)}{R}\xi_n P_n(\cos\theta). \tag{4.8}$$

Thus, allowing for (4.8), we obtain from (4.6) that in the approximation in question the nonuniformly heated droplet retains a spherical shape in its motion.

To illustrate the dependence of the force that acts on the nonuniformly heated droplet, Fig. 1 gives curves that relate the coefficients $\varphi_\mu = f_\mu/f_\mu^*$, $\varphi_q = f_q/f_q^*$, and $\varphi_d = f_d/f_d^*$ to the average surface temperature T_s for large mercury droplets of radius $R = 10^{-5}$ m that move in water at $T_\infty = 293$ K. The values of the viscosity are described accurate to 0.7% by the coefficients $A = 5.4348$, $F_1 = -1.4249$, and $F = 8.6798$; $\partial\sigma/\partial T = -2\cdot10^{-4}$ N/(m·K), Pr = 6.75; $f_\mu^* = f_\mu|_{T_s=303\mathrm{K}}$; $f_q^* = f_q|_{T_s=303\mathrm{K}}$; $f_d^* = f_d|_{T_s=303K}$.

NOTATION

μ_{liq}, dynamic viscosity of the liquid; $q_p(r, \theta)$, density of the heat sources inside the droplet, which depends on the spherical coordinates r and θ ($0 \le \theta \le \pi$); φ, azimuthal angle; σ, coefficient of surface tension of the droplet; $\mu_\infty = \mu_{\mathrm{liq}}(T_\infty)$; T_∞, temperature of the liquid away from the particle; A, const; F_n, const; T, P, ρ, c_p, and λ, temperature, pressure, density, heat capacity at constant pressure, and thermal conductivity; $\mathbf{U}$, mass velocity; $\mathbf{U}_\infty$, velocity of the plane-parallel flow of the liquid about the droplet ($\mathbf{U}_\infty \| OZ$); x_k, Cartesian coordinates; R, radius of the droplet; U_k, components of the mass velocity $\mathbf{U}$; $\mathbf{e}_r$ and $\mathbf{e}_\theta$, unit vectors of the spherical coordinate system. Subscripts and superscripts: liq and p, liquid and droplet, respectively; ∞, values of physical quantities taken at locations away from the droplet (at infinity); i, internal; e, external; d, motion; s, average.

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